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MATHS (AS)

ROYAL AIR FORCE—AIRCRAFT APPRENTICES

No. 1 Radio School, Locking

FINAL EXAMINATION IN EDUCATIONAL SUBJECTS

JANUARY 1961 (97TH) ENTRY

NOVEMBER 1962

MATHEMATICS
(Advanced Stream)

Time allowed—Three hours

Candidates are to attempt any SIX *questions*

All questions carry equal marks

[P.T.O.

1. (*a*) (i) Given that

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\text{, and}$$

$$I = \frac{ER}{R^2 + \omega^2 L^2},$$

prove that

$$\frac{E}{I} = \frac{L}{CR}.$$

(ii) Evaluate I when $E = 2$, $R = 7{\cdot}6$, $\omega = 0{\cdot}031$ and $L = 16{\cdot}2$.

(*b*) Solve the equation

$$2\sqrt{t} - 7 + \frac{4}{\sqrt{t}} = 0$$

correct to three significant figures. (Hint: write $\sqrt{t} = x$.)

2. (*a*) The factors of $A^3 - B^3$ are $(A - B)(A^2 + AB + B^2)$. Using this solve the equation $x^3 = 1$ completely. If the two complex roots are denoted by α and β, show that $\alpha = \beta^2$.

(*b*) Simplify

(i) $(3 + 2j)(1 + j)(5 - 4j)$, and

(ii) $6(\cos 50° + j \sin 50°) \div \dfrac{1}{2(\cos 20° + j \sin 20°)}$.

3. (*a*) Evaluate without using tables

(i) $\log_{64}(\frac{1}{2})$, and

(ii) $\log_{\frac{1}{5}}(125)$.

(*b*) Simplify

(i) $\log_{10} 20 + 7 \log_{10} \dfrac{15}{16} + 5 \log_{10} \dfrac{24}{25} + 3 \log_{10} \dfrac{80}{81}$, and

(ii) $\log \dfrac{15}{16} + \log \dfrac{64}{81} - \log \dfrac{4}{27}$.

(*c*) Solve the equation

$$6^x = 126.$$

4. (*a*) Given that $y = x^2$, find $\dfrac{dy}{dx}$ from first principles.

(*b*) Differentiate with respect to x

(i) $y = (x^2 + 3x)(x^2 - x + 2)$,

(ii) $y = \dfrac{2x}{(1 - x)^2}$,

(iii) $y = x^2 \cos^2 (2x)$, and

(iv) $y = [\log_e (\sin 5x)] \, . \, e^{\cos x}$.

(*c*) The distance s ft of an object moving from a fixed point, is given by

$$s = 3t^3 + 4t^2 + 6$$

after a time t.

Calculate

(i) the velocity of the object after 4 secs, and

(ii) the time taken to acquire a velocity of 100 ft/sec.

5. (*a*) Find the first maximum and minimum values of

$$y = \sin x + \cos x.$$

(*b*) Show that $y = ae^{-pt}$ is a solution of the differential equation

$$\frac{d^2y}{dt^2} - p\frac{dy}{dt} = 2p^2y.$$

6. (*a*) Solve the following equations for

$$0° < x° < 360°$$

(i) $\sin 2x = \sin x$, and

(ii) $2 \sin^2 2x + \cos 2x = 0$.

(*b*) In a triangle the lengths of the sides are 73 ft, 82 ft, and 91 ft. Find the angle contained between the sides of length 82 ft and 91 ft.

[P.T.O.

7. (*a*) Integrate the following:

(i) $\tan \frac{\theta}{2} \, d\theta$,

(ii) $\sin^5 x \, . \cos x \, . \, dx$, and

(iii) $\frac{dx}{e^x}$.

(*b*) Evaluate

(i) $\int_0^2 (x + 1)^2 \, dx$, and

(ii) $\int_0^{\pi/3} (2 \cos x + 3 \sin x) \, dx$.

8. (*a*) Find the mean value of the expression

$$y = (3x + 2)^2$$

between the values $x = 1$ and $x = 3$.

(*b*) Given that $\sinh x = \left(\frac{e^x - e^{-x}}{2}\right)$ and $\cosh x = \left(\frac{e^x + e^{-x}}{2}\right)$ show that

(i) $\int \sinh x \, dx = \cosh x$, and

(ii) $\int \cosh x \, dx = \sinh x$.

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